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We examine several proposed schemes by Franson et al. for quantum logic gates based on non-local exchange interactions between two photons in a medium. In these schemes the presence of a *single* photon in a given mode is claimed to induce a large phase shift on another photon propagating in the same medium. We conclude that: (i) the scheme using collisional effects [Franson, PRL **78**, 3852 (1997)] is flawed and cannot work; (ii) the recent scheme based on the Dicke cooperative mechanism [Franson et al., quant-ph/9912121] is physically sound, and is a striking, ingenious application of the cooperatively enhanced single-photon absorption and emission known for excitons in solids. Notwithstanding the chances of realizing the intriguing cooperative mechanism, the authors have not shown that it can yield the conditional phase shift required for a quantum logic gate.

I. INTRODUCTION

The fact that photons almost do not interact with each other limits our ability to build photonic logical gates for two and more qubits. Nonlinear effects whereby one light beam influences another require large numbers of photons or else photon confinement in high-Q cavities [1,2]. Therefore a gate in which one photon-qubit would influence another is difficult to construct. This difficulty has motivated Franson et al. [3–6] to search for a new effect that would enable the creation of a simple gate operating at the two-photon level.

Their basic proposal for the photonic computer is introduced in [4]. Each qubit is represented by a photon which can travel along two alternate paths in an interferometer. Single qubit manipulation can be easily performed by setting proper phases in the relevant interferometer. The reading of the computed result is done by photodetection. The initial state should be prepared as a multi-mode product of single-photon Fock states. The authors of [4] propose using quantum non-demolition photon number measurements to select such states from a larger set of coherent states.

The essential and most difficult part of the proposed scheme is an efficient two-bit quantum gate. Physically, the phase which one photon picks up in an interferometer should be determined by the path which the other photon chooses in another interferometer. For this to occur both interferometers should share one common branch in a nonlinear medium, such that when both photons travel along this branch an extra phase equal to π would arise. This would require a giant cross-Kerr effect with negligible photon loss. The main advantage of such a scheme would be experimental simplicity, as compared to cavity-based schemes [1,2].

Here we examine the main assumptions and results of the schemes detailed in Ref. [3–6]. In Sec. II an overview of these schemes is outlined. In Sec. III we present our detailed evaluation. Our conclusions are given in Sec. IV.

II. OVERVIEW OF NON-LOCAL PHOTON EXCHANGE SCHEMES

The underlying model of [3,4] can be described as follows. Two light pulses simultaneously enter a medium of N off-resonant atoms. The two pulses contain $n_{1,2}$ photons, respectively. Applying perturbation theory we can find the energy eigenvalues of the joint system of atoms and photons (within the dipole and rotating-wave approximation). Some of the fourth-order perturbation terms contain products of the photon numbers $n_1 n_2$. Such terms can be characterized by Feynman diagrams in which a photon from pulse 1 is virtually absorbed by atom A and re-emitted by atom B, whereas a photon from pulse 2 is absorbed by atom B and re-emitted by atom A. The number of such terms is proportional to the number of various atomic pairs, i.e., $N(N-1)/2$. If such terms contributed to the total energy, the physical result would be a nonlinear refractive index of the medium with very interesting properties: (i) the non-linear term would be proportional to the number of atoms squared N^2; (ii) the index of refraction caused by pulse 1 would be proportional to the photon number in pulse 2 and vice versa. However, after summing them up, all the terms containing $n_1 n_2$ exactly cancel each other. We may ask whether such non-linear terms are just a mathematical artifact of perturbation theory or whether they correspond to some real physical situation. If the latter is true, the question is how to suppress some of the terms so that the remaining terms contribute to an experimentally observable non-linear effect.

Franson's first suggestion for suppressing some of the $n_1 n_2$ terms was to take advantage of collisional line broadening [3]. The model used N two-level atoms and the resulting non-linear part of the total atom-photon energy was claimed to be

$$\Delta E \approx -\frac{2M^4N^2n_1n_2f_R}{\delta^3}\frac{w^2}{(\delta_1-\delta_2)^2}, \tag{1}$$

where M is the transition matrix element, f_R (< 1) is a factor taking into account decoherence due to a possible which-way information about the position of the photon absorption and re-emission, $\delta_{1,2} \approx \delta$ are the detunings of the modes 1 and 2 from the atomic resonance, and w is the collisional line-width. For an efficient non-linear coupling between single-photon pulses one should find a sufficiently dense medium (N large) and a broad collisional line-width, so that $w^2/(\delta_1 - \delta_2)^2 \approx 1$, which would yield a considerable non-linear energy shift.

The next suggestion was to manipulate the atomic resonance frequencies [5,6]. The authors considered three-level atomic media, where strong laser pulses coupled to one of the atomic transitions would manipulate the resonance frequency of another transition, e.g., by Stark shifts. Thus, the medium would be turned on and off resonance with the incident photons, which would be absorbed and re-emitted in a controlled way.

In Ref. [6] it is assumed that the photonic states would be coupled to collective N-atom excitations (Dicke states). The coupling frequency is then proportional to $\sqrt{N}$ and is assumed to be larger than any decay and decoherence rates. In both schemes with external driving pulses the authors claim that the photon of one kind would exhibit Rabi oscillations whose frequency depends on the presence or absence of the photon of the other kind. A proper choice of the external strong laser pulses is then claimed to effectively induce the required non-linear coupling of two single-photon pulses.

III. DETAILED EXAMINATION OF THE SCHEMES

A. Collisional scheme

In the derivation of the results of [3] there are several unjustified assumptions:

(1) The model of [3] assumes two weak off-resonant light pulses propagating in a medium of two-level atoms. It is supposed that the effective Hilbert space describing the system is spanned by quantum states with different numbers of photons in the two relevant optical modes (1 and 2) and with different excited and de-excited atoms. All other optical modes are ignored: processes where photons can be re-emitted to modes other than 1 and 2 are disregarded. This is an arbitrary assumption: in open space photons are re-emitted into a continuum of modes, so that the main effect would be scattering. This would, of course, invalidate the potential application of the proposed effect as a quantum gate.

(2) Even if we go along with the model where only two optical modes are present, we cannot accept the main result Eq. (1). Its derivation is based on replacing in the fourth-order perturbation expansion the energy levels ϵ_m which are influenced by collisions by the complex values $\epsilon_m - iw$. However, in doing so, we always obtain *zero* for the non-linear n_1n_2 terms. The only way to obtain Eq. (1) is to assume that the states with *no excited atoms* ($n_1 \pm 1$ photons in mode 1 and $n_2 \mp 1$ in mode 2) suffer from collisional decoherence and their energy levels should be modified by adding the iw term. Of course, this assumption is not physically justified.

(3) Finally, even under the unlikely assumption that *states with all atoms in the ground level decohere by collisions*, the non-linear term (1) would be accompanied by an imaginary part

$$\Delta E' = -2if_RM^4N^2n_1n_2\frac{w}{\delta^2(\delta_1-\delta_2)^2}, \tag{2}$$

whose magnitude is *larger* by a factor of δ/w than the real part. Hence, decay would always dominate any such non-linear phase shift and render the effect unobservable.

B. Laser-induced nonlinear phase shift in "ladder" systems

In the scheme of [5] one applies strong laser pulses which induce AC Stark shifts and thereby change the detuning of the near-resonant atomic transition from the relevant single-photon-carrying modes. Again, this model assumes that after photons 1 and 2 are absorbed by the atoms (not only virtually but also really, when the atoms are on resonance), they can only be re-emitted into the useful modes 1 and 2. Thus, it is assumed in [5] that in *open space*, a single optical photon on resonance with the atomic medium can *perform a Rabi oscillation without being scattered to the continuum of other modes*. In other words, a resonant atom which absorbs the only photon from the traveling field would re-emit it to *exactly the same* (now empty) mode. In Ref. [5], no mechanism has been presented that would justify the assumed mode selectivity.

C. The Dicke cooperative mechanism in Raman transitions

A mechanism supporting mode selectivity and the elimination of the mode continuum is presented in the e-print [6]. It is argued there that a field mode of the wavevector $\mathbf{k}$ is effectively only coupled to a particular superposition of atomic excited states (Dicke state [7]), namely

$$|p(\mathbf{k})\rangle = \frac{1}{\sqrt{N}}\sum_j \exp(i\mathbf{k}\cdot\mathbf{r}_j)|e_j\rangle, \tag{3}$$

where $|e_j\rangle$ determines the state with jth atom excited and all the other atoms being in the ground state; the summation runs over all N atoms. The coupling between the field mode and the corresponding Dicke state is $\sqrt{N}M$, M being the coupling between the field and a single atom. In an original extension of the standard formalism in [6] the same cooperative enhancement is shown to apply to Raman transitions: The Dicke collective state will then be excited for the ground state by the operator

$$\hat{R}_+(\vec{k}-\vec{K}) = \sum_{j=1}^{N} \hat{R}_+^{(j)} e^{(\vec{k}-\vec{K})\cdot\vec{r}_j} \tag{4}$$

where $\vec{k}$ and $\vec{K}$ are the wavevectors of the incident photon and the external laser field, respectively, and $\hat{R}_+^{(j)}$ is the jth atom raising operator.

The spectacular feature of the Dicke formalism is that, in the absence of decoherence or losses, *any distribution* of atomic positions is guaranteed to have a state of maximal cooperation, such that the corresponding transition rate (Rabi frequency) is enhanced by $\sqrt{N}$ compared to that of a single atom. For Raman transitions we estimate that the cooperatively enhanced Rabi frequency is (in SI units)

$$\Omega_{\rm Raman}^{\rm (coop)} \approx \sqrt{\rho\omega/(\epsilon_0\hbar)}\mu\Omega_0/\Delta \tag{5}$$

where ρ is the atomic density, ω the transition frequency, ϵ_0 the vacuum permittivity, μ the transition dipole moment, Δ the detuning from the single-photon resonance and $\Omega_0 = \mu E/\hbar$ is the strong-field Rabi frequency.

The crux of the effect is that during the relevant time, energy can only be exchanged between two states - a photon in a single field mode and a single-excitation atomic collective state (3), thus exhibiting Rabi oscillations. The decay and decoherence rates leading to scattering into other modes could be presumably slower, due to the $\sqrt{N}$ proportionality of the Rabi frequency. Because of linearity and the weak dependence of the Rabi frequency on the photonic frequency ω, the same would be valid for photonic wavepackets. Under these assumptions, any sufficiently dense medium of resonant (or Raman-resonant) atoms could be transparent for single-photon light pulses: the photon would travel inside the medium "dressed" by the atomic excitations.

Evaluation

(i) The cooperative Dicke effect discussed in [6] is essentially the well-known excitonic enhancement of absorption and emission in crystals [8] except that Raman transitions are discussed in [6] instead of the standard direct transitions.

(ii) Thus far, single-photon Rabi oscillations associated with cooperative (excitonic) enhancement have only been observed in semiconducting *cavities* [9], where they have the character of the single-mode Tavis-Cummings [10] cooperative effect known for atoms in high-Q cavities [11].

(iii) By contrast, single-photon cooperative Rabi oscillations *in open space* of mode continuum suggested in [6] have never been observed. The reason is that decoherence usually prevails, i.e., is *faster* than the achievable Rabi oscillation. This can be clarified using the estimate (5) in the two possible regimes:

(a) In the *high-density* regime, $\rho/k^3 \gg 1$, corresponding to interatomic distances $r_{ij} \sim \rho^{-1/3}$ much smaller than the photon wavelength, the dominant source of decoherence detrimental to cooperation are resonant dipole-dipole interactions [12] whose rate is $\Omega_{\rm dip} \sim \gamma/(kr_{ij})^3$, γ being the radiative linewidth (for direct transitions), i.e., $\Omega_{\rm dip}$ scales as ρ/k^3. For both direct transitions and Raman transitions Eq. (5) typically yields a lower rate than the dipole-dipole rate. Only for spatially *symmetric* atomic arrangements cooperative effects prevail over the dipole-dipole dephasing [13]. For example, for interatomic distances of 10 nm $\rho \sim 10^{18}$ cm^{-3}, $\Omega_{\rm Raman}^{\rm (coop)} \lesssim 10^{12}$ s$^{-1} \lesssim \Omega_{\rm dip} \sim 10^6\gamma$.

(b) In the *low-density* regime $\rho/k^3 \lesssim 1$, the dipole-dipole rate is less than γ and does not have to hamper cooperation. However, other sources of dephasing set T_2 to be shorter than the cooperative Rabi period (typically $\Omega_{\rm Raman}^{\rm (coop)} \lesssim 10^6$ s^{-1}) both in thermal gases ($T_2 \lesssim 10^{-10}$ s) and in semiconductors ($T_2 \lesssim 10^{-12}$ s). Only at very low temperatures and specially designed materials there is some hope to observe single-photon cooperative effects.

(iv) The most important, ingenious assumption in [6] concerns the initial condition for the atom-field system: the atoms are suddenly switched on resonance in the vicinity of the photon wavepacket, which is already within the medium. This is achieved by an appropriate geometry in which the photonic wavepacket overlaps with the strong fast-switching laser pulse. Thus, only a photon being initially within the resonant medium could perform Rabi oscillations with the corresponding Dicke state. This is in contrast with the usual situation when a photon arrives at the medium which already has been resonant: the photon is then reflected or absorbed at the medium boundary, but cannot enter inside.

D. Two-photon entanglement and conditional phase shifts

Notwithstanding the chances of realizing the cooperative effect discussed above, the question is whether this effect can be used to produce the required conditional phase shift of the photonic states. In [5,6] the authors argue that a proper sequence of external laser pulses would accomplish this task. Concrete suggestions for the pulse sequences were given in [5] (five-pulse sequence), and in [6] (three-pulse sequence). In the following we discuss these two suggestions separately.

1. Five-pulse sequence

The first pulse brings the medium into resonance with photon 1 causing a π Rabi transition: if photon 1 was initially present, it is absorbed creating a single-excitation Dicke state. The second pulse brings the medium into resonance with photon 2. The Rabi frequency now depends on whether the medium is excited or not. The authors of [5] assume that the second pulse causes a 2π transition if there is no initial excitation (photon 1 is absent) or a $\sqrt{2}2\pi$ transition if there is initial medium excitation (photon 1 is present). In the latter case it is assumed that a superposition of two states is produced: a state with two atomic excitations and no photon, and a state with a single atomic excitation and a single photon in mode 2. The third pulse is used to produce a phase-shift in this superposition, and the remaining two pulses reverse the evolution of the first two pulses, to a state with the initial number of photons.

Our objection is that during the second pulse, also a state with two photons in mode 2 can be produced (as can be seen from the Hamiltonian, Eq. (31) of [5]): stimulated photon emission would occur with the same rate as photon absorption. Evolution into such a state is neglected without any justification. Of course, the presence of such a two-photon state would invalidate the function of a quantum gate, where qubits are represented by single-photon states.

2. Three-pulse sequence

The above flaw is removed in the scheme of [6]: during the second pulse in the case of initially one atomic excitation and one photon in mode 2, the system oscillates between this state and the superposition of the state with two atomic excitations and no photons and the state with two photons and no atomic excitations. The frequency of this oscillation is twice as large as the Rabi frequency in the absence of the initial atomic excitation. Thus, if there was no photon in mode 1, the photon 2 exhibits a 2π Rabi transition during the second pulse, whereas if there was a photon in mode 1, the system exhibits a 4π Rabi transition. A 2π Rabi transition returns the original state with additional sign -1, whereas a 4π Rabi transition simply reproduces the original state. The authors of [6] claim that this difference in sign produces the required conditional phase shift.

Let us investigate in more detail the evolution of the states, denoting the basis of the logical gate as $|0,0\rangle$, $|0,1\rangle$, $|1,0\rangle$, and $|1,1\rangle$. Here, e.g., $|1,0\rangle$ means that photon 1 goes through the medium whereas photon 2 does not, etc. The evolution is then as follows. (a) State $|0,0\rangle$: there are no Rabi transitions, therefore $|0,0\rangle \to |0,0\rangle$. (b) State $|0,1\rangle$: no change during the first pulse (photon 1 is absent), a 2π Rabi transition during the second pulse and no change during the third pulse, therefore $|0,1\rangle \to -|0,1\rangle$. (c) State $|1,0\rangle$: a π transition during the first pulse, a 2π transition during the second pulse and a π transition during the third pulse, therefore $|1,0\rangle \to |1,0\rangle$. (d) State $|1,1\rangle$: a π transition during the first pulse, a 4π transition during the second pulse and a π transition during the third pulse, therefore $|1,1\rangle \to -|1,1\rangle$. Thus, the transformation matrix is

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{6}$$

which is *not* the required conditional phase shift. This transformation *does not* entangle the two photons and is thus not suitable for building quantum logical gates.

IV. CONCLUSION

In the works [3–6] we have not found a convincing proof that the suggested mechanisms could produce the conditional phase shift required for a quantum gate. On the other hand, as workers in quantum optics, we find the idea of coupling the photonic state to an atomic Dicke state by fast switching, which would result in single-photon Rabi oscillations, very interesting and ingenious, even though the prospects for its realization are presently unclear.

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